

MAT 142: College Mathematics

Lecture Notes: Chapter 13

13.1: Introduction to Counting Methods

If you roll two dice, how many different pairs of numbers can appear on the upturned faces?

A. Count Sets by Creating an Organized List

B. Count by Creating a Tree Diagram

Examples:

1. Let set $S = \{A, B, C, D, E\}$. List all the ways you can select two different members from S . The order in which you select the members is NOT important.
2. How many different two-digit numbers can you form using the digits 1, 2, 5, 7, 8, and 9 with repetition? (for example, 55 is allowed)
3. You are offering guests at your party options for a three course meal. You have 3 different appetizers, 4 main courses and 2 deserts. How many different options can guests choose for their three courses?

13.2: The Fundamental Counting Principle (the FCP)

FCP: If you want to perform a series of tasks and the first task can be done in a different ways, the second can be done in b different ways, the third can be done in c different ways, and so on, then all the tasks can be done in $a \times b \times c$ etc... different ways.

Examples: Solve same the same three examples from 13.1 by using the FCP.

1. Let set $S = \{A, B, C, D, E\}$. List all the ways you can select two different members from S . The order in which you select the members is NOT important.
2. How many different two-digit numbers can you form using the digits 1, 2, 5, 7, 8, and 9 with repetition? (for example, 55 is allowed)
3. You are offering guests at your party options for a three course meal. You have 3 different appetizers, 4 main courses and 2 deserts. How many different options can guests choose for their three courses?

Using slot diagrams:

1. To open a personal locker at the health club, a person must enter four digits in order from the set 0, 1, 2, ..., 9. How many different keypad patterns are possible if:
 - a. Any digit can be used in any position and repetition of digits is allowed?
 - b. The digit 0 cannot be used as the first digit and the number must be even, but otherwise any digit can be used and repetition is allowed?
 - c. Any digit can be used in any position, but repetition is not allowed?
2. In a certain state, license plates currently consist of two letters followed by three digits. How many such license plates are possible? If the state department of transportation decides to change the plates to have three letters followed by two digits, how many plates will now be possible?

13.3: Permutations and Combinations

Two special kinds of counting:

Permutations: The number of ways that “n” items can be chosen “r” at a time.
Order IS Important!

Example: You have 8 numbers to pick from to make a 3 number (no repeats allowed) combination. How many possible combinations are there (order IS important)?

Factorials:

Notation:

Combinations: The number of ways that “n” items can be chosen “r” at a time.
Order IS NOT Important

Example: If you are playing “The Pick” lottery in AZ, you can choose six numbers from 1 to 44 without repetition (the order in which the numbers are chosen is NOT important). How many possible ticket combinations are there?

Notation:

Examples: Specify the number of ways to perform the task described. Give your answers using $P(n, r)$ or $C(n, r)$ notation and then the numeric answer.

1. Mulder will read the files on three unsolved cases from a list of seventeen. He does not care about the order in which he reads the files.
2. The manager of a baseball team has already chosen the first, second, fourth, and ninth batters in the batting lineup. He has selected five others to play and wants to write their names in the batting lineup.
3. Ten magazines are competing for three identical awards for excellence in journalism. No magazine can receive more than one award.
4. NASA wants to appoint two men and three women to become residents of an orbiting space station. The finalists for these positions consist of six men and eight women. In how many ways can NASA make this selection?